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III Semester B.A./B.Sc. Examination, December - 2019

(CBCS) (Semester Scheme) (F+R) (2015-16 and Onwards)

MATHEMATICS - III

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** questions.*

PART - A

Answer **any five** questions. **5x2=10**

1. (a) Write the order of the elements of the group (Z_4, t_4).

(b) Find all right cosets of the subgroup {0, 3} in (Z_6, t_6).

(c) Show that the sequence $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.

(d) State Cauchy's root test for convergence.

(e) Test the convergence of the series :

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.....\infty$$

(f) Evaluate $\lim\limits_{x\to\infty} x.\sin\left(\frac{1}{x}\right)$.

(g) State Cauchy's mean value theorem.

(h) Evaluate $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$.

PART - B

Answer **one** full question. **1x15=15**

2. (a) If a and b are any two arbitrary elements of a group G, then prove that $O(a)=O(b^{-1}ab)$.

(b) If G is a group of fourth roots of unity and H is a subgroup of G, where $H=\{1, -1\}$ then write all cosets of H in G. Verify Lagrange's theorem.

(c) State and prove Fermat's theorem in groups.

OR

3. (a) If a is a generator of a cyclic group G then prove that a^{-1} is also a generator.

(b) In a group G, if $O(a)=n$, $\forall a\in G$, $d=(n, m)$, then prove that $O(a^m)=\frac{n}{d}$.

(c) If G is a finite group and H is a subgroup of G then prove that order of H divides the order of G.

P.T.O.

PART - C

2x15=30

Answer **two** full questions.

4. (a) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, prove that $\lim_{n\to\infty} a_n.b_n = ab$.

(b) Discuss the nature of the sequence $\{n^{1/n}\}$

(c) Test the convergence of

(i) $n[\log(n+1) - \log n]$

(ii) $1 + \cos n\pi$

OR

5. (a) Prove that a monotonic decreasing sequence which is bounded below is convergent.

(b) Show that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2.

(c) Examine the convergence of the sequence :

(i) $\left\{\frac{1+(-1)^n n}{(n+1)}\right\}$

(ii) $(2n+3)\sin\left(\frac{\pi}{n}\right)$

6. (a) Discuss the nature of the geometric series $\sum_{n=0}^{\infty} x^n$

(b) Test the convergence of the series :

$1 + \frac{1}{2} + \frac{1\cdot3}{2\cdot4} + \frac{1\cdot3\cdot5}{2\cdot4\cdot6} + \dots\dots$

(c) Sum the series to infinity

$\frac{1}{7} - \frac{1\cdot4}{7\cdot14} + \frac{1\cdot4\cdot7}{7\cdot14\cdot21} - + \dots\dots$

OR

7. (a) State and prove Raabe's test for the convergence of series of positive terms.

(b) Discuss the Leibnitz test on alternating series $\sum(-1)^{n-1}a_n$

(c) Sum the series to infinity $\sum_{n=1}^{\infty} \frac{(n+1)(2n+1)}{(n+2)!}$

PART - D

Answer **one** full question. **1x15=15**

8. (a) State and prove Lagrange's mean value theorem.

(b) Test the differentiability of $f(x)=\begin{cases}1-3x, & x\le 1\\ x-3, & x>1\end{cases}$ at $x=1$.

(c) Expand $\log_e(1+\cos x)$ upto the term containing x^4 by using Maclaurin's series.

OR

9. (a) Prove that a function which is continuous in closed interval takes every value between its bounds atleast once.

(b) Expand sin$x$ in powers of $\left(x-\frac{\pi}{2}\right)$ by using Taylor's series expansion. Hence find the value of sin91° correct to 4 decimal places.

(c) Evaluate : (i) $\lim_{x\to\frac{\pi}{2}} \frac{\log(\sin x)}{\left(\frac{\pi}{2}-x\right)^2}$

(ii) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$

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